

The study of the nonleptonic two body B decays involving a light tensor meson in final states

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Abstract

The nonleptonic two body $B_{u,d,s,c}$ decays involving a light tensor meson in final states are studied in the perturbative QCD approach based on k_T factorization. The decay modes with a tensor meson emitted, are prohibited in naive factorization, since the emission diagram with a tensor meson produced from vacuum is vanished. While contributions from the so-called hard scattering emission diagrams and annihilation type diagrams are important and calculable in the perturbative QCD approach. The branching ratios of most decays are in the range of 10^{-4} to 10^{-8} , which are bigger by 1 or 2 orders of magnitude than the predictions given by naive factorization, but consistent with the predictions from QCD factorization and the recent experimental measurements. We also give the predictions for the direct CP asymmetries, some of which are large enough for the future experiments to measure. We also find that, even with a small mixing angle, the mixing between f_2 and f_2' can bring remarkable changes to both branching ratios and the direct CP asymmetries for some decays involving $f_2^{(\prime)}$ mesons. For decays with a vector meson and a tensor meson in final states, we predict a large percentage of transverse polarization contributions due to the contributions of the orbital angular momentum of the tensor mesons.

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I. INTRODUCTION

The two body hadronic decays of B meson are important, since they can provide constraints of the standard model Cabibbo-Kobayashi-Maskawa (CKM) matrix, a test of the QCD factorization, information on the decay mechanism and the final state interaction, and also a good place to study CP violation and new physics signal. Most of the studies concentrate on the $B \rightarrow PP$, PV and VV decays, since they are easy to be measured at experiments. Recently, more and more experimental measurements about B decays with a light tensor meson involved in the final states have been obtained[1]. Inspired by these experiments, many theoretical studies on the $B_{u,d,s,c}$ decays involving a light tensor meson have been done based on naive factorization [2–12], QCD factorization (QCDF) [13, 14] and perturbative QCD (PQCD) approach [15–18].

The tensor meson emission diagrams are prohibited, because the amplitude proportional to the matrix element $\langle T | j^\mu | 0 \rangle$ vanishes from Lorentz covariance considerations, where j^μ donates the $(V \pm A)$ current or $(S \pm P)$ density [5, 6, 13, 14]. Thus, these decays modes with a tensor meson emitted are prohibited in naive factorization. On the other hand, the naive factorization can not give creditable predictions for those color-suppressed or penguin dominant decays. What is more, the naive factorization can not deal with the pure annihilation type decays. The recent developed QCDF approach [13, 14] and the PQCD factorization approach [16–18] overcome these shortcomings by including the large hard scattering contributions and the annihilation type contributions to give more reliable predictions for these decays, especially those with tensor meson emitted or pure annihilation type decays. It is worth of mentioning that the annihilation type diagrams can be perturbatively calculated without parametrization in PQCD approach [19, 20]. Only the PQCD approach have successfully predicted the pure annihilation type decays $B_s \rightarrow \pi^+\pi^-$ [21, 22] and $B^0 \rightarrow D_s^- K^+$ [19, 23], which have been confirmed by experiments later [1, 24]. So, for those annihilation diagram dominant or pure annihilation type decays, for example, $B_c \rightarrow D^{(*)}T$ decays, the calculation in PQCD approach is more reliable than other approaches.

The studied tensor mesons include the isovector $a_2(1320)$, the isodoublet $K_2^*(1430)$, and isosinglet $f_2(1270)$ and $f_2'(1525)$ [1]. For the tensor meson with $J^P = 2^+$, both the orbital angular momentum L and the total spin S of the quark pair are equal to 1. However, their

production property in B decays is quite similar to the light vector mesons [15]. Because of the Bose statistics, the light-cone distribution amplitudes of tensor mesons are antisymmetric under the interchange of momentum fractions of the quark and anti-quark in the flavor SU(3) limit [13, 14].

B meson decays into tensor mesons are of prime interest in several aspects. The branching ratios and CP asymmetries are helpful to inspect those different theoretical calculations. Some decay modes, like $B \rightarrow K_2^*(1430)\omega$, possess a large isospin violation[25]. Moreover, from our computations, polarizations of the final state mesons in B decays into tensor mesons and vector mesons are also beyond the naive hierarchy, which is similar to some decays to two light vector mesons and shed light on the helicity structure of the electroweak interactions.

II. FORMALISM

It is well known that the key step to predict two-body hadronic $B_{(s,c)}$ decays is calculating the transition matrix elements:

$$\mathcal{M} \propto \langle h_1 h_2 | \mathcal{H}_{eff} | B_{(s,c)} \rangle \quad (1)$$

with the weak effective Hamiltonian \mathcal{H}_{eff} written as [26]

$$\begin{aligned} \mathcal{H}_{eff} = & \frac{G_F}{\sqrt{2}} \left\{ V_{u(c)b}^* V_{u(c)X} [C_1(\mu) O_1^q(\mu) + C_2(\mu) O_2^q(\mu)] \right. \\ & \left. - V_{tb}^* V_{tX} \left[\sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right\}, \end{aligned} \quad (2)$$

with the CKM matrix elements $V_{u(c)b(X)}$ and $V_{tb(X)}$ ($X = d, s$). $c_i(\mu)$ are the effective Wilson coefficients at the renormalization scale μ , whose expression can be found at ref.[27]. The local four-quark operators O_j ($j = 1, \dots, 10$) can be given as

$$O_1^u = (\bar{b}_\alpha u(c)_\beta)_{V-A} (\bar{u}(\bar{c})_\beta X_\alpha)_{V-A}, \quad O_2^q = (\bar{b}_\alpha u(c)_\alpha)_{V-A} (\bar{u}(\bar{c})_\beta X_\beta)_{V-A}, \quad (3)$$

for the current-current (tree) operators,

$$O_3 = (\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \quad O_4 = (\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A}, \quad (4)$$

$$O_5 = (\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \quad O_6 = (\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A}, \quad (5)$$

for the QCD penguin operators, and

$$O_7 = \frac{3}{2}(\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} e_{q'}(\bar{q}'_\beta q'_\beta)_{V+A}, \quad O_8 = \frac{3}{2}(\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A}, \quad (6)$$

$$O_9 = \frac{3}{2}(\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} e_{q'}(\bar{q}'_\beta q'_\beta)_{V-A}, \quad O_{10} = \frac{3}{2}(\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A}, \quad (7)$$

for the electro-weak penguin operators, with the SU(3) color indices α and β and the active quarks $q' = (u, d, s, c, b)$ at the scale m_b . The combinations of the Wilson coefficients are defined as [28]:

$$\begin{aligned} a_1 &= C_2 + C_1/3, & a_2 &= C_1 + C_2/3, \\ a_i &= C_i + C_{i\pm 1}/3, & i &= 3, 5, 7, 9/4, 6, 8, 10. \end{aligned} \quad (8)$$

In the B meson rest frame, the light final state mesons with large momenta are moving fast. The spectator quark in B meson is soft in the initial state, while collinear in the final state. There must be a hard gluon to kick the soft spectator quark into collinear and energetic one. Thus the process is perturbatively calculable. The basic idea of PQCD approach is keeping the intrinsic transverse momentum k_T of valence quarks in the hadrons. The end-point singularity in collinear factorization can be avoided. On the other hand, the double logarithms, which are caused by the additional energy scale introduced by the transverse momentum, can be resummed through the renormalization group equation to result in the Sudakov form factor. This form factor effectively suppresses the end-point contribution of distribution amplitude of mesons in the small transverse momentum region, which makes the calculation in the PQCD approach reliable and consistent.

In hadronic $B_{(s,c)}$ decays, there are several typical energy scales and expansions with respect to the ratios of the scales are usually carried out, for example, the W boson mass scale, the b quark mass scale, and the factorization scale $\sqrt{\bar{\Lambda}m_B}$ with $\bar{\Lambda} \equiv m_B - m_b$. We can perturbatively calculate the electroweak physics higher than W boson mass scale. The hard dynamics from m_W scale to m_b scale can be included in the so-called Wilson coefficients in eq.(2) by using the renormalization group equation. The physics between M_B scale and the factorization scale can be calculated perturbatively and included in the so-called Hard Kernel in the PQCD approach. The soft dynamics below the factorization scale is nonperturbative and described by the hadronic wave functions of mesons, which is universal for all decay modes. Finally, based on the factorization, the decay amplitude can be described as the

following convolution of the the Wilson coefficients $C(t)$, the hard scattering kernel and the light-cone wave functions $\Phi_{h_i,B}$ of mesons [29],

$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \times Tr \left[C(t) \Phi_B(x_1, b_1) \Phi_{h_2}(x_2, b_2) \Phi_{h_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right], \quad (9)$$

where Tr denotes the trace over Dirac and colour indices, b_i is the conjugate variable of quark's transverse momentum k_{iT} , x_i is the momentum fractions of valence quarks and t is the largest energy scale in the hard part $H(x_i, b_i, t)$. The jet function $S_t(x_i)$ smears the end-point singularities on x_i , which is from the threshold resummation of the double logarithms $\ln^2 x_i$ [30]. The Sudakov form factor $e^{-S(t)}$ from the resummation of the double logarithms suppresses the soft dynamics effectively i.e. the long distance contributions in the large b region [31, 32].

For a tensor meson, the polarization tensor $\epsilon_{\mu\nu}(\lambda)$ with helicity λ can be expanded through the polarization vectors $\epsilon^\mu(0)$ and $\epsilon^\mu(\pm 1)$ [13, 14]

$$\begin{aligned} \epsilon^{\mu\nu}(\pm 2) &\equiv \epsilon(\pm 1)^\mu \epsilon(\pm 1)^\nu, \\ \epsilon^{\mu\nu}(\pm 1) &\equiv \sqrt{\frac{1}{2}} [\epsilon(\pm 1)^\mu \epsilon(0)^\nu + \epsilon(0)^\mu \epsilon(\pm 1)^\nu], \\ \epsilon^{\mu\nu}(0) &\equiv \sqrt{\frac{1}{6}} [\epsilon(+1)^\mu \epsilon(-1)^\nu + \epsilon(-1)^\mu \epsilon(+1)^\nu] + \sqrt{\frac{2}{3}} \epsilon(0)^\mu \epsilon(0)^\nu. \end{aligned} \quad (10)$$

In order to calculate conveniently, a new polarization vector ϵ_T is defined as [15]

$$\epsilon_{T\mu} = \frac{1}{m_B} \epsilon_{\mu\nu}(h) P_B^\nu, \quad (11)$$

with

$$\begin{aligned} \epsilon_{T\mu}(\pm 2) &= 0, \\ \epsilon_{T\mu}(\pm 1) &= \frac{1}{m_B} \frac{1}{\sqrt{2}} (\epsilon(0) \cdot P_B) \epsilon_\mu(\pm 1), \\ \epsilon_{T\mu}(0) &= \frac{1}{m_B} \sqrt{\frac{2}{3}} (\epsilon(0) \cdot P_B) \epsilon_\mu(0). \end{aligned} \quad (12)$$

The ± 2 polarizations do not contribute, which is consistent with the angular momentum conservation argument in B decays. The ϵ_T is similar with the ϵ of vector state, regardless of the related constants [15]. After this simplification, the wave function for a generic tensor

meson are defined by [15]

$$\begin{aligned}\Phi_T^L &= \frac{1}{\sqrt{6}} \left[m_T \not{\epsilon}_{\bullet L}^* \phi_T(x) + \not{\epsilon}_{\bullet L}^* \not{P} \phi_T^t(x) + m_T^2 \frac{\epsilon_{\bullet} \cdot v}{P \cdot v} \phi_T^s(x) \right] \\ \Phi_T^\perp &= \frac{1}{\sqrt{6}} \left[m_T \not{\epsilon}_{\bullet \perp}^* \phi_T^v(x) + \not{\epsilon}_{\bullet \perp}^* \not{P} \phi_T^T(x) + m_T i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon_{\bullet \perp}^{*\nu} n^\rho v^\sigma \phi_T^a(x) \right],\end{aligned}\quad (13)$$

with the vector $\epsilon_{\bullet\mu} \equiv \frac{\epsilon_{\mu\nu} v^\nu}{P \cdot v}$ related to the polarization tensor. The twist-2 and twist-3 distribution amplitudes are given by [13–15]

$$\begin{aligned}\phi_T(x) &= \frac{f_T}{2\sqrt{2N_c}} \phi_\parallel(x), \quad \phi_T^t = \frac{f_T^\perp}{2\sqrt{2N_c}} h_\parallel^{(t)}(x), \\ \phi_T^s(x) &= \frac{f_T^\perp}{4\sqrt{2N_c}} \frac{d}{dx} h_\parallel^{(s)}(x), \quad \phi_T^T(x) = \frac{f_T^\perp}{2\sqrt{2N_c}} \phi_\perp(x), \\ \phi_T^v(x) &= \frac{f_T}{2\sqrt{2N_c}} g_\perp^{(v)}(x), \quad \phi_T^a(x) = \frac{f_T}{8\sqrt{2N_c}} \frac{d}{dx} g_\perp^{(a)}(x),\end{aligned}\quad (14)$$

with the form

$$\begin{aligned}\phi_{\parallel,\perp}(x) &= 30x(1-x)(2x-1), \\ h_\parallel^{(t)}(x) &= \frac{15}{2}(2x-1)(1-6x+6x^2), \quad h_\parallel^{(s)}(x) = 15x(1-x)(2x-1), \\ g_\perp^{(a)}(x) &= 20x(1-x)(2x-1), \quad g_\perp^{(v)}(x) = 5(2x-1)^3.\end{aligned}\quad (15)$$

As mentioned before, based on the Bose statistics, the above light-cone distribution amplitudes of the tensor meson are antisymmetric under the interchange of momentum fractions of the quark and anti-quark in the SU(3) limit (i.e. $x \leftrightarrow 1-x$) [13, 14], which is consistent with the fact that $\langle 0 | j^\mu | T \rangle = 0$, where j^μ is the $(V \pm A)$ or $(S \pm P)$ current.

All other meson wave functions $B_{u,d,s,c}$ and π , K etc. are adopted the same as in other pQCD papers[19, 20, 23, 33, 34], since we have argued that they should be universal for all decay channels.

III. RESULTS AND DISCUSSIONS

The traditional emission diagrams with a tensor meson emitted are prohibited in naive factorization, because a tensor meson can not be produced from the local $(V \pm A)$ or tensor currents. Thus, the predictions can not be given within the naive factorization framework. The hard scattering contributions and annihilation type contributions are important to explain the large branching ratios. When the emitted meson is the traditional light meson,

TABLE I: The PQCD predictions of CP-averaged branching ratios (in units of 10^{-6}) for some $B \rightarrow PT$ decays, with the errors from hard parameters, NLO corrections and power corrections, together with Isgur-Scora-Grinstein-Wise II (ISGW2) model [7] and QCDF results [14]. The experimental data are from Ref.[35].

Decay Modes	class	This Work	ISGW2 [7]	QCDF [14]	Expt.
$B^+ \rightarrow K_2^{*0} \pi^+$	PA	$0.9^{+0.2+0.2+0.3}_{-0.2-0.2-0.2}$...	$3.1^{+8.3}_{-3.1}$	$5.6^{+2.2}_{-1.4}$
$B^+ \rightarrow f_2 K^+$	T,PA,P	12^{+3+3+3}_{-2-3-3}	0.34	$3.8^{+7.8}_{-3.0}$	$1.06^{+0.28}_{-0.29}$
$B^+ \rightarrow f_2' K^+$	P,PA	$3.8^{+0.4+0.9+1.0}_{-0.4-0.8-0.8}$	0.004	$4.0^{+7.4}_{-3.6}$	< 7.7
$B^0 \rightarrow K_2^{*+} \pi^-$	PA	$1.0^{+0.2+0.2+0.3}_{-0.2-0.2-0.2}$...	$3.3^{+8.5}_{-3.2}$	< 6.3
$B^0 \rightarrow f_2' K^0$	P,PA	$3.7^{+0.3+0.7+0.9}_{-0.4-0.8-0.9}$	0.00007	$3.8^{+7.3}_{-3.5}$...
$B^0 \rightarrow a_2^- K^+$	T,PA	$5.0^{+1.6+1.4+1.3}_{-1.4-1.1-1.0}$	0.58	$9.7^{+17.2}_{-8.1}$...
$B^+ \rightarrow a_2^0 \pi^+$	T,C	$29^{+13+14+3}_{-11-10-3}$	26.02	30^{+14}_{-12}	...
$B^+ \rightarrow a_2^+ \pi^0$	T,C	$0.3^{+0.0+0.1+0.0}_{-0.0-0.1-0.0}$	0.01	$2.4^{+4.9}_{-3.1}$...
$B^+ \rightarrow a_2^+ \eta'$	C,PA,P	$3.5^{+1.4+1.6+1.1}_{-1.0-1.1-0.8}$	13.1	$1.1^{+4.7}_{-1.2}$...
$B^+ \rightarrow f_2 \pi^+$	T	$43^{+19+19+4}_{-15-14-4}$	28.74	27^{+14}_{-12}	$15.7^{+6.9}_{-4.9}$
$B^+ \rightarrow f_2' \pi^+$	T	$1.2^{+0.3+0.4+0.1}_{-0.2-0.3-0.1}$	0.37	$0.09^{+0.24}_{-0.09}$...
$B^0 \rightarrow a_2^- \pi^+$	T	$99^{+35+43+6}_{-30-32-10}$	48.82	52^{+18}_{-18}	< 3000
$B^0 \rightarrow a_2^+ \pi^-$	T,PA	$2.7^{+0.5+0.8+0.4}_{-0.3-0.5-0.3}$...	$2.1^{+4.3}_{-1.7}$...
$B^0 \rightarrow f_2 \pi^0$	C	$2.8^{+0.7+0.7+0.6}_{-0.6-0.6-0.4}$	0.003	$1.5^{+4.2}_{-1.4}$...
$B^0 \rightarrow f_2' \pi^0$	P	$0.2^{+0.0+0.1+0.0}_{-0.0-0.1-0.0}$	4.0×10^{-5}	$0.05^{+0.12}_{-0.05}$...

the contributions from two hard scattering emission diagrams cancel with each other due to the symmetry. however, the symmetry is destroyed, when the tensor meson is emitted, because the distribution amplitudes of tensor meson are anti-symmetric. The two diagrams strengthen with each other. Thus, the hard scattering contributions are no longer negligible but sizable for the color suppressed modes with the enhancement of Wilson coefficients.

A. $B \rightarrow PT$ decays in PQCD approach

From Table.I, one can see that the CP -averaged branching ratios of $B \rightarrow PT$ decays can reach the order of 10^{-5} , which is accessible at the current experiments. The predicted branching ratios of penguin-dominated and color-suppressed decays in PQCD are larger than those of naive factorization [7–9], but are close to the QCDF predictions [14], which is caused by the fact that the naive factorization can not evaluate the contributions from

TABLE II: The PQCD predictions of direct CP asymmetries(%) for $B \rightarrow PT$ decays, comparison with the QCDF results [14]. The experimental data are from Ref.[35].

Decay Modes	This Work	QCDF [14]	Expt.
$B^+ \rightarrow K_2^{*0}\pi^+$	$-5.5^{+0.3+2.6+1.6}_{-0.4-0.0-1.2}$	$1.6^{+2.2}_{-1.8}$	5^{+29}_{-24}
$B^+ \rightarrow f_2 K^+$	-25^{+2+2+5}_{-1-3-6}	$-39.5^{+49.4}_{-25.5}$	-68.0^{+19}_{-17}
$B^+ \rightarrow K_2^{*+}\eta$	$-5.4^{+1.1+2.2+2.3}_{-0.6-2.0-1.3}$	$1.5^{+7.4}_{-5.6}$	-45 ± 30
$B^0 \rightarrow a_2^- K^+$	$-48^{+2+1+7}_{-2-0-10}$	$-21.5^{+28.9}_{-35.0}$...
$B^+ \rightarrow a_2^+ \eta$	$-91^{+8+10+12}_{-4-1-5}$	$27.6^{+73.4}_{-127.6}$...
$B^+ \rightarrow a_2^+ \eta'$	-45^{+1+1+7}_{-1-0-9}	$31.3^{+61.3}_{-131.3}$...
$B^+ \rightarrow f_2 \pi^+$	28^{+3+1+9}_{-3-1-7}	$60.2^{+27.1}_{-72.3}$	41 ± 30
$B^0 \rightarrow f_2' \eta$	$71^{+0+11+11}_{-3-15-12}$	0.0	...
$B^0 \rightarrow f_2' \eta'$	$46^{+3+14+19}_{-7-12-19}$	0.0	...

penguin operators well. On the other hand, the B to tensor form factor calculated in PQCD approach is larger than that used in QCDF [14], thus for the tree-dominated modes such as $a_2^- \pi^+$ and $f_2 \pi^+$, the predicted results are bigger than QCDF predictions [14]. Although the $B^+ \rightarrow a_2^0 \pi^+$ is also tree-dominant, the result is the same as the prediction of QCDF, which is caused by the cancellation from hard scattering contributions with a tensor emitted.

For $B^+ \rightarrow K_2^{*0} \pi^+$ and $B^0 \rightarrow K_2^{*+} \pi^-$ decays, the predictions in naive factorization approach [7] are 0 indeed, because the emitted meson is the tensor meson K_2^* . In PQCD approach, the expected hard scattering contributions are suppressed by the small Wilson coefficients. Thus the dominant contribution is from the chiral enhanced annihilation type contributions. While in QCDF, the dominant contribution comes from hard scattering diagrams with quark loop corrections, which is next-to-leading order and not considered in this work. The branching ratios of $a_2^+ \pi^-$ and $a_2^+ \pi^0$ modes are highly suppressed relative to $a_2^- \pi^+$ and $a_2^0 \pi^+$, respectively, because the expected dominant contribution of $B \rightarrow a_2^+ \pi^0 (a_2^+ \pi^-)$ is the color favored tensor emission diagram, while the dominant contribution for the other two channels is the color enhanced diagram with pion emission. Many other smaller branching ratios for $B \rightarrow PT$ decays are listed in Tables of Ref.[16].

We also summarize the direct CP asymmetries for some $B \rightarrow PT$ decays in Table II. The full results about CP violation can be found in Ref.[16]. Although some channels have very large direct CP asymmetries, they are difficult for experiments due to the small branching ratios. We recommend the experimenters to search for the direct CP asymmetry in the

channels like $B^+ \rightarrow f_2 K^+$, $B^0 \rightarrow a_2^- K^+$, $B^+ \rightarrow a_2^+ \eta'$ and $B^+ \rightarrow f_2 \pi^+$, for they have both large branching ratios and direct CP asymmetry parameters. In fact, there are already some experimental measurements shown in Table II. Although the error bars are still large, we are happy to see that all these measured entries have the same sign as our theoretical calculations. This may imply that our approach gives the leading order strong phase in these channels.

Similar to the $\eta - \eta'$ system, we have taken the $f_2 - f_2'$ mixing into account [16]. Although the mixing angle is small, the interference between f_2^q and f_2^s can bring some remarkable changes. For example, the branching ratio of $B^+ \rightarrow f_2' \pi^+$ is enhanced comparing with the prediction of QCDF [14] without the mixing. Although suppressed by the small mixing angle, the color-allowed contribution from f_2^q term, which is not considered in QCDF [14], is at the same order as that of f_2^s term. The enhancement from f_2^q term makes the branching ratio larger than the prediction without taking the mixing into account. The mixing can also bring remarkable change to direct CP asymmetry. Without the mixing, the direct CP asymmetry of $B \rightarrow f_2' \eta^{(\prime)}$ decay should be 0, since there are no contributions from penguin operators. After taking the mixing into account, the direct CP asymmetries are quite large shown in Table II, since these decays get penguin contributions from f_2^q .

B. $B_s \rightarrow PT$ decays in PQCD approach

We have also studied the two-body hadronic $B_s \rightarrow PT$ decays in the PQCD approach and given the predictions about branching ratios and CP observables [36]. Like $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ systems, the $B_s^0 - \bar{B}_s^0$ also mixes through the weak interaction. Since the mass difference ΔM between the mass eigenstates is much larger than the decay width Γ of the B_s meson, the $B_s^0 - \bar{B}_s^0$ oscillate very frequent. As a result, the measurements, such as time-dependent CP-violation parameters, are very difficult for super B factories, while are feasible in LHCb experiments.

For $\Delta S = 0$ decays, the B_s^0 (\bar{B}_s^0) meson decays to the final state f (\bar{f}), but not to \bar{f} (f) with $f \neq \bar{f}$. One can find in Table III that only the $B_s^0 \rightarrow \pi^+ K_2^{*-}$ decay has a sizable branching ratio due to the color enhanced emission amplitude T . Because the factorizable emission contributions are suppressed, those color-suppressed modes, such as $B_s^0 \rightarrow \bar{K}^0 a_2^0$, $\bar{K}^0 f_2$ and $\bar{K}^0 f_2'$, are similar with their PV partners [22]. While for the color-

TABLE III: Some results about branching ratios (10^{-7}) and direct CP asymmetries of the $\Delta S=0$ $B_s^0 \rightarrow PT$ decays.

Modes	Class	$Br(10^{-7})$	Direct A_{CP} (%)
$B_s^0 \rightarrow \pi^+ K_2^{*-}$	T	90_{-32-6}^{+40+4}	13_{-2-2}^{+2+2}
$B_s^0 \rightarrow \bar{K}^0 a_2^0$	C, PA	$2.0_{-0.3-0.3}^{+0.4+0.2}$	38_{-10-7}^{+7+6}
$B_s^0 \rightarrow \bar{K}^0 f_2$	C, PA	$3.4_{-0.6-0.7}^{+0.7+0.7}$	-24_{-6-5}^{+5+3}
$B_s^0 \rightarrow K^- a_2^+$	T, PA	$1.5_{-0.2-0.3}^{+0.3+0.4}$	39_{-1-4}^{+8+1}

TABLE IV: Some results of branching ratios (10^{-7}) and CP observables for the $\Delta S = 1$ $B_s^0 \rightarrow PT$ decays.

Modes	Class	Br	C_f	D_f	S_f	A_{CP} (%)
ηa_2^0	C, A	$0.047_{-0.010-0.012}^{+0.013+0.010}$	$0.02_{-0.02-0.06}^{+0.01+0.01}$	$0.40_{-0.01-0.04}^{+0.01+0.06}$	$0.92_{-0.01-0.03}^{+0.01+0.02}$	-3.6
ηf_2	PC	$9.8_{-2.2-2.6}^{+2.7+3.2}$	$-0.014_{-0.008-0.010}^{+0.003+0.008}$	$-0.995_{-0.001-0}^{+0.001+0.002}$	$-0.098_{-0.007-0.020}^{+0.007+0.004}$	0.30
$\eta f_2'$	PA	96_{-19-30}^{+20+36}	$0.022_{-0.004-0.003}^{+0.004+0.003}$	-1.000_{-0-0}^{+0+0}	$0.024_{-0.004-0.005}^{+0.004+0.003}$	-0.10
$\eta' a_2^0$	C, A	$0.13_{-0.03-0.03}^{+0.03+0.03}$	$0.03_{-0.01-0.01}^{+0.01+0.02}$	$0.28_{-0-0.03}^{+0.03+0.04}$	$0.96_{-0.01-0.01}^{+0+0.01}$	-3.7
$\eta' f_2$	PC	30_{-7-10}^{+7+11}	$-0.005_{-0.012-0.010}^{+0+0.002}$	$-0.994_{-0.001-0.001}^{+0.001+0.001}$	$-0.104_{-0.006-0.006}^{+0.011+0.006}$	0.40
$\eta' f_2'$	PA, PT	245_{-59-84}^{+69+99}	$-0.007_{-0.003-0.001}^{+0.004+0}$	-1.000_{-0-0}^{+0+0}	$-0.009_{-0.002-0.001}^{+0.006+0.004}$	0.030

avored $B_s^0 \rightarrow K^- a_2^+$ decay, the branching ratio 1.50×10^{-7} is much smaller than the $B_s^0 \rightarrow K^- \rho^+$ one, 1.78×10^{-5} , because the factorizable emission amplitude, which is dominant in $B_s^0 \rightarrow K^- \rho^+$, is forbidden in naive factorization with a tensor meson emission. Combining those predictions with the $B \rightarrow PT$ ones, we find large U-spin asymmetries in some decay modes, such as $B_s^0 \rightarrow \pi^+ K_2^{*-}$ and $B_s^0 \rightarrow K^- a_2^+$ [36].

For some $\Delta S = 1$ B_s^0 (\bar{B}_s^0) meson decays, whose final states are CP eigenstates, i.e. $f = \bar{f}$, the results are summarized in Table IV. One can find that $B_s^0 \rightarrow \eta' a_2^0(f_2, f_2')$ have branching ratios larger than those of the $B_s^0 \rightarrow \eta a_2^0(f_2, f_2')$ modes, because the dominant $\bar{s}s$ constituent is suppressed for decays involving η than η' . On the other hand, the $\Delta I = 1$ modes, like $B_s^0 \rightarrow \eta a_2^0$ and $\eta' a_2^0$, are highly suppressed, compared to the corresponding $\Delta I = 0$ modes, $B_s^0 \rightarrow \eta f_2$ and $\eta' f_2$, because the dominant contributions from different penguin operators are destructive in former modes, but constructive in latter decays. Contrary to the $\Delta S = 0$ decays, the direct CP violation C_f 's are tiny, because the contributions from tree operators are too small.

There exist more complicated $\Delta S = 1$ modes, in which either a B_s^0 or \bar{B}_s^0 meson can decay into f or \bar{f} with $f \neq \bar{f}$. The results are summarized in Table V. One can find that,

TABLE V: Branching ratios (in units of 10^{-7}) and CP observables for the rest $\Delta S = 1$ decays.

Modes	C_f	D_f	S_f	$C_{\bar{f}}$
$\pi^+ a_2^-$	$-0.15^{+0.01+0.02}_{-0.04-0.05}$	$-0.98^{+0+0.01}_{-0-0.01}$	$-0.10^{+0.07+0.05}_{-0.01-0.01}$	$-0.05^{+0.07+0.07}_{-0.02-0.01}$
$K^+ K_2^{*-}$	$0.49^{+0.07+0.02}_{-0.06-0.01}$	$-0.85^{+0.04+0}_{-0.03-0}$	$-0.18^{+0.02+0.03}_{-0.04-0.05}$	$0.03^{+0.11+0.09}_{-0.08-0.13}$
$K^0 \bar{K}_2^{*0}$	$0.24^{+0.08+0.03}_{-0.06-0.05}$	$-0.91^{+0.03+0.02}_{-0.02-0.02}$	$-0.34^{+0.03+0.04}_{-0.03-0.03}$	$0.24^{+0.08+0.03}_{-0.06-0.05}$
	$D_{\bar{f}}$	$S_{\bar{f}}$	Br	$A_{CP}(\%)$
$\pi^+ a_2^-$	$-0.98^{+0.01+0.01}_{-0.01-0.01}$	$0.18^{+0.04+0.04}_{-0.02-0.03}$	$1.8^{+0.4+0.6}_{-0.2-0.8}$	13^{+3+5}_{-5-5}
$K^+ K_2^{*-}$	$-0.71^{+0.09+0.03}_{-0.06-0.02}$	$-0.70^{+0.07+0.03}_{-0.07-0.03}$	86^{+20+28}_{-16-24}	-28^{+2+5}_{-3-6}
$K^0 \bar{K}_2^{*0}$	$-0.91^{+0.03+0.02}_{-0.02-0.02}$	$-0.34^{+0.03+0.04}_{-0.03-0.03}$	70^{+14+24}_{-12-20}	0

for $B_s^0 \rightarrow \bar{K}^0 K_2^{*0}$, the f -related CP observables are the same as the \bar{f} -related ones. What's more, A_{CP} is exactly zero due to the shortage of tree contributions. It is then straightforward to arrive at $\lambda_f = \bar{\lambda}_{\bar{f}}$, and thus $C(D, S)_f = C(D, S)_{\bar{f}}$ and $A_{CP} = 0$.

C. $B_{(s)} \rightarrow D^{(*)}T, \bar{D}^{(*)}T$ decays in PQCD approach

 TABLE VI: Branching ratios of some $B_{(s)} \rightarrow DT$ decays in the PQCD approach together with results from Isgur-Scora-Grinstein-Wise (ISGW) II model [9, 10] (unit: 10^{-7}).

Decay Modes	Class	This Work	SDV[9]	KLO[10]
$B^0 \rightarrow D^0 f_2$	C	$2.1^{+0.8+0.3+0.3}_{-0.7-0.3-0.2}$	0.36	...
$B^0 \rightarrow D^0 f_2'$	C	$0.038^{+0.015+0.006+0.005}_{-0.013-0.006-0.005}$	0.0071	...
$B^+ \rightarrow D^0 K_2^{*+}$	C	$37^{+14+7+5}_{-12-8-5}$	13	12
$B_s \rightarrow D^0 \bar{K}_2^{*0}$	C	$1.4^{+0.7+0.2+0.2}_{-0.6-0.2-0.2}$	0.46	...
$B^0 \rightarrow D^+ a_2^-$	T	15^{+8+2+2}_{-6-3-2}	12	...
$B^+ \rightarrow D^+ a_2^0$	T	$9.4^{+4.6+1.2+1.2}_{-3.4-1.6-1.1}$	6.5	...
$B_s \rightarrow D^+ K_2^{*-}$	T	11^{+6+1+1}_{-5-1-1}	8.3	...
$B^0 \rightarrow D_s^+ K_2^{*-}$	E	$0.61^{+0.15+0.12+0.08}_{-0.14-0.16-0.07}$
$B^+ \rightarrow D^+ K_2^{*0}$	A	$5.3^{+1.8+0.7+0.7}_{-1.7-0.7-0.7}$
$B_s \rightarrow D^0 a_2^0$	E	$3.9^{+1.4+0.7+0.5}_{-1.2-1.0-0.5}$

There are two categories of decays with one charmed meson in the final states. The $B_{(s)} \rightarrow \bar{D}^{(*)}T$ decays governed by the $\bar{b} \rightarrow \bar{c}$ transition, while the $B_{(s)} \rightarrow D^{(*)}T$ decays governed by the $\bar{b} \rightarrow \bar{u}$ transition. Clearly, there is a large enhancement of CKM matrix elements $|V_{cb}/V_{ub}|^2$ for the the former kinds of decays, especially for those without a strange quark in the four-quark operators. One can find that the branching ratios of $B_{(s)} \rightarrow \bar{D}^{(*)}T$ decays

TABLE VII: Branching ratios (unit: 10^{-7}) and the percentage of transverse polarizations R_T (unit:%) of some $B_{(s)} \rightarrow D^* T$ decays in the PQCD approach together with results from ISGW II model [9, 10].

Decay Modes	Class	Branching Ratio(10^{-7})			$R_T(\%)$
		This Work	SDV[9]	KLO[10]	
$B^0 \rightarrow D^{*0} f_2$	C	$2.7^{+1.2+0.4+0.4}_{-1.0-0.3-0.3}$	0.53	...	26^{+4+1}_{-4-1}
$B^+ \rightarrow D^{*0} K_2^{*+}$	C	$72^{+28+12+10}_{-24-9-9}$	21	19	35^{+4+1}_{-4-1}
$B_s \rightarrow D^{*0} \bar{K}_2^{*0}$	C	$2.1^{+1.0+0.3+0.3}_{-0.8-0.3-0.2}$	0.7	...	21^{+3+1}_{-4-1}
$B^+ \rightarrow D^{*+} K_2^{*0}$	A	18^{+5+0+2}_{-5-2-3}	82^{+2+4}_{-3-3}
$B^+ \rightarrow D_s^{*+} f_2'$	A	22^{+7+1+3}_{-6-2-3}	4.0	2.0	83^{+5+2}_{-5-2}
$B^+ \rightarrow D_s^{*+} \bar{K}_2^{*0}$	A	$1.3^{+0.4+0.1+0.2}_{-0.3-0.2-0.2}$	81^{+2+4}_{-2-3}
$B^0 \rightarrow D_s^{*+} K_2^{*-}$	E	$0.57^{+0.13+0.12+0.07}_{-0.13-0.11-0.07}$	12^{+2+3}_{-2-3}
$B_s \rightarrow D^{*+} a_2^-$	E	$5.4^{+1.8+1.4+0.7}_{-1.6-1.3-0.7}$	21^{+3+5}_{-3-4}

are larger than those of $B_{(s)} \rightarrow D^{(*)} T$ decays shown in Tables VI,VII and Tables VIII,IX, respectively. For most of the $B_{(s)} \rightarrow D^{(*)} T$ decays, the branching ratios are at the order 10^{-6} or 10^{-7} ; while for the $B_{(s)} \rightarrow \bar{D}^{(*)} T$ decays, the branching ratios are at the order 10^{-4} or 10^{-5} . With a charm quark in the final state, all these decays are governed by tree level current-current operators, without penguin operator contribution. Since the direct CP asymmetry is proportional to the interference between two different contributions. all these decays have no direct CP asymmetries.

Due to the symmetry, the contributions from two hard scattering emission diagrams cancel with each other in the charmless $B \rightarrow PP, PV$ decays, thus the hard scattering emission diagrams are suppressed heavily. However, when the emitted meson is the $D(\bar{D})$ or tensor meson, the cancellation is weakened or removed. The symmetry of those two hard scattering emission diagrams is broken by the big difference between $c(\bar{c})$ quark and the light quark in the heavy $D(\bar{D})$ meson. As stated above, when the tensor meson is emitted, the contributions from two hard scattering diagrams strengthen with each other, because the wave function of tensor meson is antisymmetric under the interchange of the momentum fractions of the quark and antiquark. As a result, for those color suppressed decay modes, the hard scattering contribution plays the crucial role in the decay amplitude, since the factorizable contributions are suppressed by the small Wilson coefficient a_2 . Therefore, the predicted branching ratios in the PQCD approach are larger than those of naive factorization

TABLE VIII: Branching ratios of some $B_{(s)} \rightarrow \bar{D}T$ decays calculated in the PQCD approach together with results from ISGW II model [9, 10] (unit: 10^{-5}).

Decay Modes	Class	This Work	SDV[9]	KLO[10]
$B^0 \rightarrow \bar{D}^0 a_2^0$	C	$12_{-3}^{+3+3+1}_{-3-0}$	8.2	4.8
$B^0 \rightarrow \bar{D}^0 f_2$	C	$9.5_{-2.3}^{+2.5+3.6+0.5}_{-3.7-0.3}$	8.8	5.3
$B^+ \rightarrow \bar{D}^0 a_2^+$	T,C	$42_{-13}^{+17+13+2}_{-14-1}$	18	10
$B_s \rightarrow \bar{D}^0 \bar{K}_2^{*0}$	C	$20_{-6}^{+8+4+1}_{-5-1}$	11	...
$B^0 \rightarrow D^- a_2^+$	T	$40_{-13}^{+15+13+2}_{-12-1}$
$B^0 \rightarrow D^- K_2^{*+}$	T	$1.2_{-0.4}^{+0.5+0.5+0.1}_{-0.5-0.1}$
$B_s \rightarrow D_s^- a_2^+$	T	$11_{-4}^{+6+6+1}_{-5-0}$
$B^0 \rightarrow D_s^- K_2^{*+}$	E	$6.1_{-1.7}^{+1.7+0.4+0.3}_{-1.0-0.2}$
$B_s \rightarrow D^- a_2^+$	E	$0.23_{-0.08}^{+0.08+0.02+0.01}_{-0.04-0.01}$

[9, 10]. Taking $B^0 \rightarrow \bar{D}^0 f_2$ as example, the PQCD prediction $\mathcal{B}(B^0 \rightarrow \bar{D}^0 f_2) = 9.46 \times 10^{-5}$, which is larger than other approaches, agrees better with the experimental data $(12 \pm 4) \times 10^{-5}$ [35]. For those decays with a tensor meson emitted, since the factorizable contributions are prohibited, the naive factorization can not give predictions [9, 10]. But these decays can get contributions from hard scattering and annihilation type diagrams. The PQCD approach can calculate these contributions and give the predictions for the first time in table VI-IX. Our results show that the contributions from annihilation diagrams are even at the same order as the emission diagrams in some decay modes. The interference between contributions from emission diagram and contributions from annihilation diagrams can explain why $\mathcal{B}(B^0 \rightarrow \bar{D}^{(*)0} a_2^0) > \mathcal{B}(B^0 \rightarrow \bar{D}^{(*)0} f_2)$, which is similar to $\mathcal{B}(B^0 \rightarrow \bar{D}^{(*)0} \rho^0) > \mathcal{B}(B^0 \rightarrow \bar{D}^{(*)0} \omega)$ [37].

For those color allowed decay channels, since the contribution of hard scattering diagrams is highly suppressed by the Wilson coefficient, the decay amplitude is dominated by the contribution from factorizable emission diagrams with the Wilson coefficient a_1 , which can be naively factorized as the product of the Wilson coefficient a_1 , the decay constant of D meson and the B to tensor meson form factor. So our predictions basically agree with the predictions of naive factorization approach in Ref.[9]. The difference is caused by parameter changes and the interference from suppressed hard scattering and annihilation type diagrams.

For $B \rightarrow D^*(\bar{D}^*)T$ decays, we also calculate the percentage of transverse polarizations, which are listed in Table VII and Table IX (extracted from ref.[17]). From our computa-

TABLE IX: Branching ratios (unit: 10^{-5}) and the percentage of transverse polarizations R_T (unit:%) of color-suppressed $B_{(s)} \rightarrow \bar{D}^* T$ decays in PQCD approach together with results from ISGW II model [9, 10].

Decay Modes	Class	Branching Ratio(10^{-5})			$R_T(\%)$
		This Work	SDV[9]	KLO[10]	
$B^0 \rightarrow \bar{D}^{*0} a_2^0$	C	$39^{+14+2+2}_{-11-1-1}$	12	7.8	73^{+5+9}_{-4-8}
$B^0 \rightarrow \bar{D}^{*0} f_2$	C	$38^{+14+2+2}_{-12-1-1}$	13	8.4	70^{+6+9}_{-6-6}
$B^0 \rightarrow \bar{D}^{*0} K_2^{*0}$	C	$5.3^{+1.7+0.8+0.3}_{-1.4-0.7-0.2}$	1.3	1.1	71^{+2+9}_{-2-9}
$B_s \rightarrow \bar{D}^{*0} f_2'$	C	$5.0^{+2.1+0.4+0.3}_{-1.7-0.5-0.2}$	1.1	...	71^{+1+8}_{-1-7}
$B_s \rightarrow \bar{D}^{*0} \bar{K}^{*0}$	C	$70^{+29+4+4}_{-24-6-2}$	17	...	68^{+2+9}_{-1-8}

tions, we find that some decays disobey the naive factorization assumption. For those color suppressed $B \rightarrow \bar{D}^* T$ decays with the \bar{D}^* emitted, the transverse contributions are dominant about 70%. The \bar{c} is right-handed; while the u quark is left-handed, because they are produced through $(V - A)$ current. Thus, the \bar{D}^* meson is longitudinally polarized. But the massive \bar{c} quark can flip easily from right handed to left handed. As a result, the polarization of the \bar{D}^* meson becomes transverse with $\lambda = -1$. On the other hand, because of the additional contribution of orbital angular momentum, the recoiled tensor meson can also be transversely polarized with $\lambda = -1$ easily. So the transverse polarization fractions can be as large as 70%. While for color suppressed $B \rightarrow D^* T$ decays with D^* meson emitted, the percentage of transverse polarizations are only at the range of 20% to 30%. the emitted D^* meson can also be transversely polarized, but with $\lambda = +1$. According to the angular momentum conservation, the recoiled tensor meson must be also transversely polarized with $\lambda = +1$. This calls for the tensor meson getting contributions from both orbital angular momentum and spin, which is symmetric. Because the distribution amplitude of tensor meson is anti-symmetric, the total wave function is anti-symmetric, which is forbidden by Bose statistics. So the transverse polarization of final states is suppressed.

For the W annihilation(A) type $B_{(s)} \rightarrow D^* T$ decays, we also find very large transverse polarizations up to 80% . The light quark and anti-quark produced through hard gluon are left-handed or right-handed with equal opportunity. The c quark is left-handed, and then the D^* meson can be longitudinally polarized, or be transversely polarized with $\lambda = -1$. For the tensor meson, the anti-quark from weak interaction is right-handed; while the quark

produced from hard gluon can be either left-handed or right-handed. When taking into account the additional orbital angular momentum, the tensor meson can be longitudinally polarized or transversely polarized with $\lambda = -1$. So the transverse contributions can become so large with interference from other diagrams. On the other hand, the W exchange(E) diagrams can not contribute large transverse contributions, which is consist with the argument in $B \rightarrow D^*V$ decays in refs.[23, 38].

Some decays involving $f_2^{(\prime)}$ in the final states provide a potential way to measure the mixing angle θ of f_2 and f_2' . For example, $B^0 \rightarrow D^0 f_2^{(\prime)}$, the branching ratios have the simple relation

$$r = \frac{\mathcal{B}(B \rightarrow D f_2')}{\mathcal{B}(B \rightarrow D f_2)} = \frac{\sin^2 \theta}{\cos^2 \theta}, \quad (16)$$

since the $s\bar{s}$ term dose not contribute. In our calculation, it shows $r \simeq 0.02$ with $\theta = 7.8^\circ$.

D. $B_c \rightarrow D^{(*)}T$ decays in PQCD approach

The B_c meson is unique, since it is a heavy quarkonium with two different flavors. Either the heavy b quark or the c quark can decay individually. And the W annihilation diagram decay of the B_c meson is also important due to the large CKM matrix element V_{cb} . Since the c quark in the $B_c \rightarrow D^{(*)}T$ decays is the spectator quark, these decays are dominated by the $B_c \rightarrow D^{(*)}$ transition form factors with a tensor meson emitted from vacuum, which are prohibited in naive factorization. To our knowledge, these decays are never considered in theoretical papers due to this difficulty of factorization. In order to give the predictions to these decay channels, it is necessary to go beyond the naive factorization to calculate the hard scattering and annihilation-type diagrams. What is more, the annihilation amplitudes will be dominant in these $B_c \rightarrow D^{(*)}T$ decays, because they are proportional to the large CKM matrix elements $V_{cb}V_{cs(d)}^*$ instead of $V_{ub}V_{us(d)}^*$ in the emission diagrams. Some of these decays are pure W annihilation type.

Compared with the $B_c \rightarrow D^0 \rho^+$ decay in table III in ref.[39], which is dominated by the color allowed emission diagram, the corresponding $B_c \rightarrow D^0 a_2^+$ decay has a smaller branching ratio, for the emission of a tensor meson is prohibited. For the penguin dominant decay channels $B_c \rightarrow D^0 K^{*+}$ and $D^+ K^{*0}$, with $b \rightarrow s$ transition, due to the enhancement of the large CKM elements $V_{cs(d)}$ and large Wilson coefficient a_1 , the annihilation diagrams are

at the same order magnitude as penguin emission diagrams but with a minus sign shown in table III of ref.[39]. While the corresponding $B_c \rightarrow D^0 K_2^{*+}$ and $D^+ K_2^{*0}$ decays have much larger branching ratios shown in table X, without the cancellation from emission diagram. With additional transverse contributions, the B_c meson decays to tensor and D^* meson final states, have branching ratio as large as 10^{-4} in Table XI that will be easier to search for experiments.

TABLE X: Branching ratios (10^{-6}) and direct CP asymmetries (%) for some $B_c \rightarrow DT$ decays calculated in the PQCD approach .

Decay Modes	Class	Br(10^{-6})	$A_{CP}^{dir}(\%)$
$B_c \rightarrow D^0 a_2^+$	A	$2.2^{+0.8+0.2+0.2}_{-0.7-0.2-0.2}$	$6.47^{+1.35+5.33+0.00}_{-1.15-1.59-0.74}$
$B_c \rightarrow D^0 K_2^{*+}$	A	$31^{+10+3+1}_{-9-3-1}$	$-0.44^{+0.13+0.10+0.10}_{-0.15-0.22-0.02}$
$B_c \rightarrow D^+ K_2^{*0}$	A	$32^{+11+3+1}_{-10-2-1}$	0.0
$B_c \rightarrow D_s^+ f_2'$	A	$41^{+12+4+1}_{-11-4-1}$	$-0.11^{+0.02+0.03+0.02}_{-0.02-0.06-0.00}$

The LHC experiment, specifically the LHCb, can produce around 5×10^{10} B_c events each year[40, 41]. The B_c decays with a decay rate at the level of 10^{-6} can be detected with a good precision at LHC experiments [42]. So some of these $B_c \rightarrow D^{(*)}T$ decays can be observed in the experiments. For example, $B_c \rightarrow D^{(*)+} K_2^{*0}$, the branching ratio is at the order of $10^{-5}(10^{-4})$. Taking into account the branching ratios of D^+ and K_2^{*0} decays with charged final states (10% ($D^+ \rightarrow K^- \pi^+ \pi^+$)[43] and 25% ($\mathcal{B}(K_2^{*0} \rightarrow K\pi) = (49.9 \pm 1.2)\%$) respectively) and assuming a total efficiency of 1% [43], one can expect about dozens of events every year. For $B_c \rightarrow D_s f_2'$ with branching ratio 4×10^{-5} , taking into account the

TABLE XI: Branching ratios (10^{-6}), direct CP asymmetries (%) and the percentage of transverse polarizations $R_T(\%)$ for some $B_c \rightarrow D^*T$ decays calculated in the PQCD approach.

Decay Modes	Class	Br(10^{-6})	$A_{CP}^{dir}(\%)$	$R_T(\%)$
$B_c \rightarrow D^{*0} K_2^{*+}$	A	$151^{+30+18+5}_{-27-11-3}$	$-0.15^{+0.02+0.05+0.03}_{-0.02-0.08-0.06}$	82.5
$B_c \rightarrow D^{*+} K_2^{*0}$	A	$158^{+31+16+0}_{-29-15-13}$	0.0	80.3
$B_c \rightarrow D_s^{*+} \bar{K}_2^{*0}$	A	$8.9^{+1.7+0.8+0.5}_{-1.6-0.9-0.3}$	$2.3^{+0.2+0.9+0.0}_{-0.1-0.5-0.0}$	82.0
$B_c \rightarrow D_s^{*+} f_2'$	A	$190^{+31+20+6}_{-28-13-4}$	$-0.036^{+0.004+0.011+0.008}_{-0.003-0.012-0.001}$	89.5

branching ratios of D_s^+ and f_2' decays with charged final states (6% ($D_s^+ \rightarrow K^+ K^- \pi^+$) and 45% ($\mathcal{B}(f_2' \rightarrow K \bar{K}) = (88.7 \pm 2.2)\%$) respectively) and assuming a total efficiency of 1%, one can expect about one hundred events every year. They are the most promising channels to be measured in the current experiments.

Due to the same reason in W annihilation type $B_{(s)} \rightarrow D^* T$ decays, for the W annihilation diagrams dominant decays, we also predict the percentages of the transverse polarization around 80% or even bigger. The CP averaged branching ratios, direct CP asymmetries, and the transverse polarization fractions for all the decays of the type $B_{(s)} \rightarrow D^* T$ decays can be found in ref [18].

IV. SUMMARY

We have studied the $B_{u,d,s,c}$ decays involving a light tensor meson in final states within the framework of perturbative QCD approach. We calculate the contributions of different diagrams, especially the hard scattering and annihilation type diagrams, which are important to explain the large experimental data and the large transverse polarization fractions. For some decays with tensor meson emitted or pure annihilation type decays, we give the predictions for the first time. For those penguin dominant decays and color-suppressed decays, we give larger and more reliable predictions, which agree the experimental data better. For those color suppressed $B_{(s)} \rightarrow \bar{D}^* T$ decays, the transversely polarized contributions from hard scattering diagrams are very large. For those W annihilation type $B_{(c)} \rightarrow D^* T$ decays, the transverse polarized contributions from factorizable annihilation diagrams are as large as 80%.

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